25. The sum of the lengths of the three sides of a right-angled triangle is equal to 18 and the sum of the squares of the lengths of the three sides is equal to 128. What is the area of the triangle?
(A) 18 (B) 16 (C) 12 (D) 10 (E) 9

26. You are given 5 boxes, 5 black and 5 white balls. You choose how to put the balls in the boxes (each box has to contain at least one ball). Your opponent comes and draws one ball from one box of his choice and he wins if he draws a white ball. Otherwise, you win. How should you arrange the balls in the boxes to have the best chance to win?
(A) You put one white and one black ball in each box.
(B) You arrange all the black balls in three boxes, and all the white balls in two boxes.
(C) You arrange all the black balls in four boxes, and all the white balls in one box.
(D) You put one black ball in every box, and add all the white balls in one box.
(E) You put one white ball in every box, and add all the black balls in one box.

27. Nine integers are written in the cells of a 3 × 3 table. The sum of the nine numbers is equal to 500. It is known that the numbers in any two neighboring cells (that is cells sharing a common side) differ by 1. What is the number in the central cell?

	?	

(A) 50 (B) 54 (C) 55
(D) 56 (E) 57

28. If $|x| + x + y = 5$ and $x + |y| - y = 10$, what is the value of $x + y$?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

29. How many three-digit positive integers ABC exist, such that $(A + B)^C$ is a three-digit integer and an integer power of 2?
(A) 15 (B) 16 (C) 18 (D) 20 (E) 21

30. Each of the 2017 people living on an island is either a liar (and always lies) or a truth-teller (and always tells the truth). More than one thousand of them take part in a banquet, all sitting together at a round table. Each of them says: "Of the two people beside me, one is a liar and the other one a truth-teller". How many truth-tellers are there on the island at most?
(A) 1683 (B) 668 (C) 670 (D) 1344 (E) 1343

Laiks uzdevumu risināšanai – 75 minūtes!

Starptautiskā konkursa
„Ķenguru"
uzdevumi

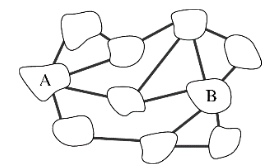
23.03.2017. 11.-12. klases

3 point problems

1. $\dfrac{20 \cdot 17}{2+0+1+7} =$
(A) 3.4 (B) 17 (C) 34 (D) 201.7 (E) 340

2. Ben likes to play with his H0-model railroad. He modeled some things in the H0-ratio 1: 87, even a 2 cm high model of his brother. What is the real height of his brother?
(A) 1.74 m (B) 1.62 m (C) 1.86 m (D) 1.94 m (E) 1.70 m

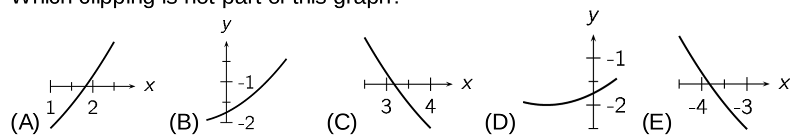
3. In the figure we see 10 islands that are connected by 15 bridges. What is the smallest number of bridges that can be eliminated in order to make it impossible to get from A to B by bridge?



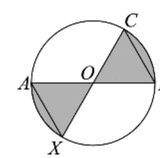
(A) 1 (B) 2 (C) 3
(D) 4 (E) 5

4. Two positive numbers a and b are such that 75% of a equals 40% of b. This means that
(A) $15a = 8b$ (B) $7a = 8b$ (C) $3a = 2b$ (D) $5a = 12b$ (E) $8a = 15b$

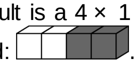
5. Four of the following five clippings are part of the graph of the same quadratic function. Which clipping is not part of this graph?

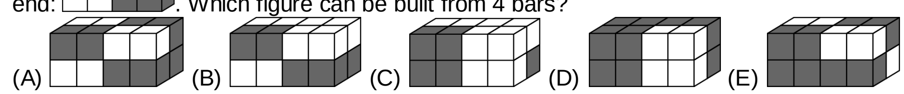


6. Given a circle with center O and diameters AB and CX such that $OB = BC$. What portion of the area of the circle is shaded?

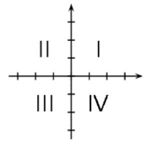


(A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{7}$ (D) $\frac{3}{8}$ (E) $\frac{4}{11}$

7. A bar consists of 2 white and 2 gray cubes glued together such that the result is a 4 × 1 × 1 bar with 2 white cubes on one end and 2 gray cubes on the other end: . Which figure can be built from 4 bars?



8. Which quadrant contains no points of the graph of the linear function $f(x) = -3.5x + 7$?



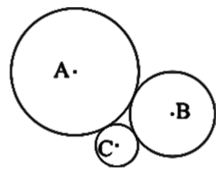
(A) I (B) II (C) III
(D) IV (E) All quadrants contain points.

9. Each of the following five boxes are filled with red and blue balls as labeled. Ben wants to take one ball out of the boxes without looking. From which box should he take the ball to have the highest probability to get a blue ball?
(A) 10 blue, 8 red (B) 6 blue, 4 red (C) 8 blue, 6 red (D) 7 blue, 7 red (E) 12 blue, 9 red

10. The graph of which of the following functions has the most points in common with the graph of the function $y = x$?
(A) $y = x^2$ (B) $y = x^3$ (C) $y = x^4$ (D) $y = -x^4$ (E) $y = -x$

4 point problems

11. Three mutually tangent circles with centres A, B, C have the radii 3, 2 and 1, respectively. What is the area of the triangle ABC?

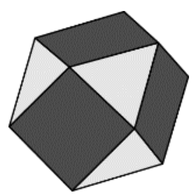


(A) 6 (B) $4\sqrt{3}$ (C) $3\sqrt{2}$
(D) 9 (E) $2\sqrt{6}$

12. The positive number p is less than 1, the number q is greater than 1. Which of the following numbers is the largest?
(A) $p \cdot q$ (B) $p + q$ (C) $\frac{p}{q}$ (D) p (E) q

13. Two right cylinders A and B have the same volume. The radius of the base of B is 10 % larger than that of A. How much larger is the altitude of A than that of B?
(A) 5 % (B) 10 % (C) 11 % (D) 20 % (E) 21 %

14. The faces of the polyhedron shown are either triangles or squares. Each square is surrounded by 4 triangles and each triangle is surrounded by 3 squares. If there are 6 squares, how many triangles are there?

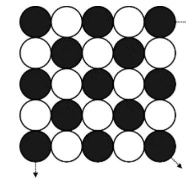


(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

15. We have four tetrahedral dice, perfectly balanced, with their faces numbered 2, 0, 1 and 7. If we roll all four of these dice, what is the probability that we can compose the number 2017 using exactly one of the three visible numbers from each dice?
(A) $\frac{1}{256}$ (B) $\frac{63}{64}$ (C) $\frac{81}{256}$ (D) $\frac{3}{32}$ (E) $\frac{29}{32}$

16. The polynomial $5x^3 + ax^2 + bx + 24$ has integer coefficients a and b. Which of the following is certainly not a root of the polynomial?
(A) 1 (B) -1 (C) 3 (D) 5 (E) 6

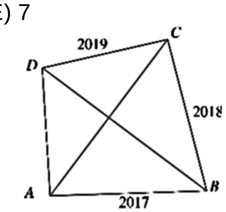
17. Julia has 2017 chips: 1009 of them are black and the rest are white. She places them in a square pattern as shown, beginning with a black chip in the upper left hand corner, alternating colours in each row and each column. How many chips of each colour are left after she has completed the largest possible square?



(A) None (B) 40 of each (C) 40 black ones and 41 white ones
(D) 41 of each (E) 40 white ones and 41 black ones

18. Two consecutive numbers are such that the sums of the digits of each of them are multiples of 7. At least how many digits does the smaller number have?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. In a convex quadrilateral $ABCD$ diagonals are perpendicular. The sides have lengths $AB = 2017$, $BC = 2018$ and $CD = 2019$ (figure not to scale). What is the length of AD?



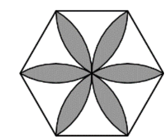
(A) 2016 (B) 2018 (C) $\sqrt{2020^2 - 4}$
(D) $\sqrt{2018^2 + 2}$ (E) 2020

20. Tytti tries to be a good little Kangaroo, but lying is too much fun. Therefore, every third thing she says is a lie and the rest is true. Sometimes she starts with a lie and sometimes with one or two true statements. Tytti is thinking of a 2-digit number and is telling her friend about it: "One of its digits is a 2. It is larger than 50. It is an even number." And: "It is less than 30. It is divisible by three. One of its digits is a 7." What is the sum of the digits of the number Tytti is thinking of?
(A) 10 (B) 12 (C) 13 (D) 15 (E) 17

5 *point problems*

21. How many positive integers have the property that the number obtained by deleting the last digit is equal to 1/14 of the original number?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

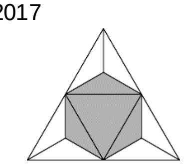
22. The picture shows a regular hexagon with side lengths equal to 1. The flower was constructed with sectors of circles of radius 1 and centers in the vertices of the hexagon. What is the area of the flower?



(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $2\sqrt{3} - \pi$ (D) $\frac{\pi}{2} + \sqrt{3}$ (E) $2\pi - 3\sqrt{3}$

23. Consider the sequence a_n with $a_1 = 2017$ and $a_{n+1} = \frac{a_n - 1}{a_n}$. Then $a_{2017} =$
(A) -2017 (B) $\frac{-1}{2016}$ (C) $\frac{2016}{2017}$ (D) 1 (E) 2017

24. Consider a regular tetrahedron. Its four corners are cut off by four planes, each passing through the midpoints of three adjacent edges (see figure). What part of the volume of the original tetrahedron is the volume of the resulting solid?



(A) $\frac{4}{5}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

11.-12. klases