

21.03.2024.

9.-10. klases

3 point problems

23. The sum of the digits of the number N is three times the sum of the digits of the number $N + 1$. What is the smallest possible sum of the digits of N ?

- (A) 9 (B) 12 (C) 15 (D) 18 (E) 27

24. Jill has some black, gray, and white unit cubes. She uses 27 of them to build a $3 \times 3 \times 3$ cube. She wants the surface to be exactly one-third black, one-third gray, and one-third white. The smallest possible number of black cubes she can use is A and the largest possible number of black cubes she can use is B . What is the value of $B - A$?

- (A) 1 (B) 3 (C) 6 (D) 7 (E) 9

25. Ann rolled a normal die 24 times. All numbers from 1 to 6 came up at least once. The number 1 came up more times than any other number. Ann added up all the numbers. The total she obtained was the largest one possible. What total did she obtain?

- (A) 83 (B) 84 (C) 89 (D) 90 (E) 100

26. Olga walked in the park. She walked half of the total time at a speed of 2 km/h. She walked half of the total distance at a speed of 3 km/h. She walked the rest of the time at a speed of 4 km/h. For what fraction of the total time did she walk at a speed of 4 km/h?

- (A) $\frac{1}{14}$ (B) $\frac{1}{12}$ (C) $\frac{1}{7}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

27. Ali wants to remove some of the integers from 1 to 25 and then separate the remaining numbers into two groups so that the products of the integers in each group are equal. What is the smallest number of integers Ali could remove?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

28. Twenty points are equally spaced on the circumference of a circle. David draws all the possible chords that connect pairs of these points. How many of these chords are longer than the radius of the circle but shorter than its diameter?

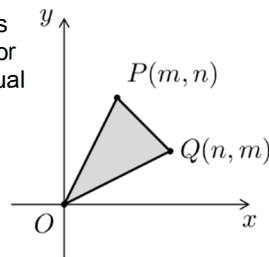
- (A) 90 (B) 100 (C) 120 (D) 140 (E) 160

29. There are n distinct lines on the plane, labeled ℓ_1, \dots, ℓ_n . The line ℓ_1 intersects exactly 5 other lines, the line ℓ_2 intersects exactly 9 other lines, and the line ℓ_3 intersects exactly 11 other lines. Which of the following is a smallest possible value of n ?

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

30. Suppose m and n are integers with $0 < m < n$. Points $P(m, n)$, $Q(n, m)$, and $O(0, 0)$ are the vertices of triangle OPQ . For how many pairs of m and n will the area of triangle OPQ be equal to 2024?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

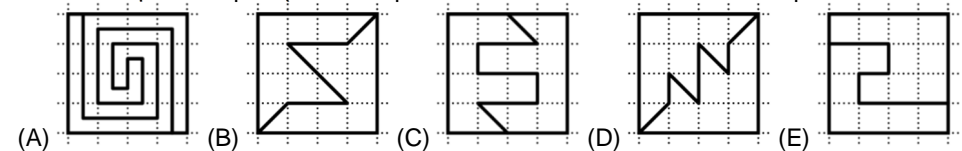


Laiks uzdevumu risināšanai – 75 minūtes!

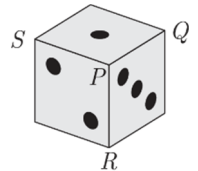
1. What is the value of $\frac{2 \cdot 0,24}{20 \cdot 2,4}$?

- (A) 0.01 (B) 0.1 (C) 1 (D) 10 (E) 100

2. Which square is split up into two pieces that do not have the same shape?



3. The number of the dots on opposite faces of a die is 7 in total. The vertex labelled P on the die is formed by the faces which have 1, 2 and 3 dots on them. Its vertex sum is the sum of the number of dots on those faces which meet at a given vertex. The vertex sum of P is $1+2+3=6$. What is the maximum of the vertex sums of vertices Q , R and S ?



- (A) 7 (B) 9 (C) 10 (D) 11 (E) 15

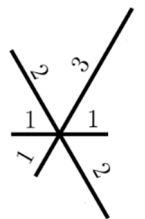


4. A hopping game is played in the following way: Each player hops into the squares, swapping between left foot - both feet - right foot - both feet - left foot - both feet, and so on, as shown. Maya played the game and hopped into exactly 48 squares starting with her left foot. How many times did her left foot touch the ground?

- (A) 12 (B) 24 (C) 36 (D) 40 (E) 48

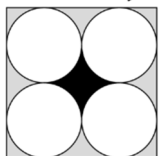
5. Tim wants to draw the figure shown without lifting his pencil off the paper. The lengths of the segments are given in the figure. He can choose to start his drawing anywhere. What is the smallest sum of the lengths of all the segments that Tim must draw to get the figure?

- (A) 14 (B) 15 (C) 16 (D) 17 (E) 18



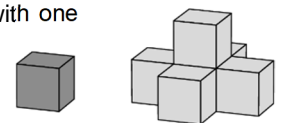
6. The figure shows a square with four circles of equal area, each touching two sides of the square and two other circles. What is the ratio between the areas of the black region and the grey region?

- (A) 1:4 (B) 1:3 (C) 2:3 (D) 3:4 (E) 2:5



7. John makes a sequence of structures on a table, beginning with one cube. He makes the next structure by adding five cubes which hide the visible faces of the initial cube, as shown. What is the smallest number of cubes he needs to add to the second structure so that all the visible faces of the second structure are hidden?

- (A) 8 (B) 9 (C) 10 (D) 13 (E) 19

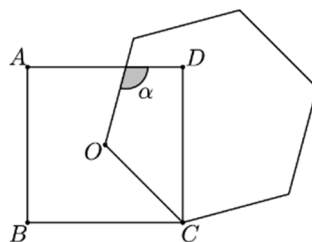


8. A three-digit palindrome is a number of the form aba where the digits a and b can either be the same or different. What is the sum of the digits of the largest three-digit palindrome that is also a multiple of 6?

- (A) 16 (B) 18 (C) 20
(D) 21 (E) 24

9. Martin draws a square with vertices A, B, C, D and a regular hexagon with side OC , where O is the center of the square. What is the size of angle α ?

- (A) 105° (B) 110° (C) 115°
(D) 120° (E) 125°



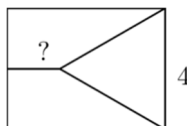
10. Ardal encloses a rectangular field with 40 m of fence. The side-lengths of the field are all prime numbers. What is the maximum possible area of the field?

- (A) $99 m^2$ (B) $96 m^2$ (C) $91 m^2$ (D) $84 m^2$ (E) $51 m^2$

4 point problems

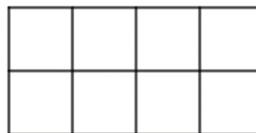
11. A rectangle is divided into three regions of equal area. One of the regions is an equilateral triangle with side-length 4 cm, the other two are trapezia, as shown in the figure. What is the length of the smaller of the parallel sides of the trapezia?

- (A) $\sqrt{2}$ cm (B) $\sqrt{3}$ cm (C) $2\sqrt{2}$ cm (D) 3 cm (E) $2\sqrt{3}$ cm



12. Yelena places the capital letters A, B, C and D into the 2×4 table. Exactly one letter is placed in each cell. She wishes to make sure that in each row and in each 2×2 square, each of the four letters appears exactly once. In how many ways can she do this?

- (A) 12 (B) 24 (C) 48 (D) 96 (E) 198



13. Dan cuts out three circles from three different pieces of coloured card. He places them on top of each other, as shown in Figure 1. He then moves the circles so that all three circles are tangent to each other, as shown in Figure 2. In the first figure, the area of the visible black region is seven times the area of the white circle. What is the ratio between the areas of the visible black regions in the two figures?

- (A) 3:1 (B) 4:3 (C) 6:5 (D) 7:6 (E) 9:7



Fig 1



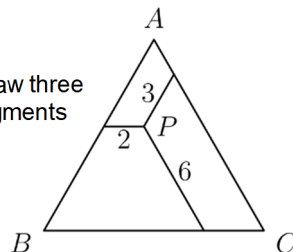
Fig 2

14. Mary's daughter gave birth to a baby girl today. In two years' time, the product of the ages of Mary, her daughter and her granddaughter will be 2024. Mary's and her daughter's ages are both even numbers. What is Mary's age now?

- (A) 42 (B) 44 (C) 46 (D) 48 (E) 50

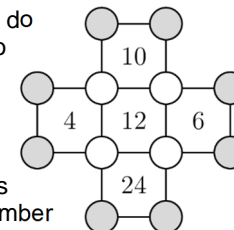
15. A point P is chosen inside an equilateral triangle. From P we draw three segments parallel to the sides, as shown. The lengths of the segments are 2, 3 and 6. What is the perimeter of the triangle?

- (A) 22 (B) 26 (C) 33 (D) 39 (E) 44



16. A number is written in each of the twelve circles shown, but we do not see these numbers. The number inside each square is equal to the product of the numbers written in circles at its four vertices. What is the product of the numbers in the eight grey circles?

- (A) 20 (B) 40 (C) 80 (D) 120 (E) 480



17. There are four vases on the table in which a number of sweets have been placed. The number of sweets in the first vase is the number of vases that contain one sweet. The number of sweets in the second vase is equal to the number of vases that contain two sweets. The number of sweets in the third vase is equal to the number of vases that contain three sweets. The number of sweets in the fourth vase is equal to the number of vases that contain zero sweets. How many sweets are in all the vases together?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

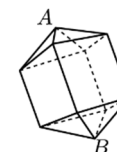
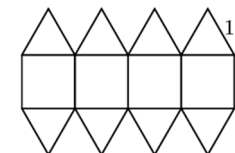
18. Jean-Philippe has n^3 ($n > 2$) identical small cubes. He used these to make a large cube and painted the entire outer surface of the large cube. The number of small cubes with only one face painted is equal to the number of those with no face painted. What is the value of n ?

- (A) 4 (B) 6 (C) 7 (D) 8 (E) 10

19. Cristina has a set of cards numbered 1 to 12. She places eight of them at the vertices of an octagon so that the sum of every pair of numbers at opposite ends of an edge of the octagon is a multiple of 3. Which numbers did Cristina not place?

- (A) 1, 5, 9, 12 (B) 3, 5, 7, 9 (C) 1, 2, 11, 12 (D) 5, 6, 7, 8 (E) 3, 6, 9, 12

20. Otis makes a net using a combination of squares and equilateral triangles, as shown in the figure. The side-length of each square and of each triangle is 1 cm. He folds the net up into the 3D shape shown. What is the distance between the vertices A and B ?



- (A) $\sqrt{5}$ cm (B) $(1 + \sqrt{2})$ cm (C) $\frac{5}{2}$ cm (D) $(1 + \sqrt{3})$ cm (E) $2\sqrt{2}$ cm

5 point problems

21. The prime factorisation of the number $n! = 1 \cdot 2 \cdot \dots \cdot n$ is of the form shown in the diagram. The primes are written in increasing order. Ink has covered some of the primes and some of the exponents. What is the exponent of 17?

- 2 . 3 . 5 . 7 . 11 . 13⁴ . 17 43 . 47
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

22. Carl always tells the truth or always lies on alternate days. One day, he made exactly four of the following five statements. Which one could he not have made on that day?

- (A) I lied yesterday and I will lie tomorrow.
(B) I am telling the truth today and I will tell the truth tomorrow.
(C) 2024 is divisible by 11.
(D) Yesterday was Wednesday.
(E) Tomorrow will be Saturday.